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Quantitative Literacy for Pre-service Elementary Teachers Within Social and Historical Contexts

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ABSTRACT

Concern with elementary teachers' subject knowledge in mathematics and science has been extensively documented in the literature over the past two decades in both mathematics and science education. In addition, there is evidence that many students begin teacher education displaying misconceptions in both mathematics and science. There is general agreement that these students' misconceptions are acquired during their school experiences and that negative attitudes contribute to poor classroom teaching which in turn contributes to poor pupil attitudes, beliefs and performance outcomes. If these pupils go on to become teachers, a cycle of negativity may be created and that if change is to occur, it must come through suitable intervention at the tertiary level. It is therefore important to make the most efficient use of the limited time available to improve the general mathematical and scientific competencies of pre-service primary teachers. This paper will outline an attempt at appropriate intervention by the implementation of an integrated unit, Quantitative Literacy, which has been developed by the author. The unit consists of an integration of topics from mathematics and science in which mathematical and scientific thinking, beliefs, and problem solving are examined entirely within social and historical contexts. This paper will present some of the integrated historical topics of the unit.

The Background to this Study

Concern with primary teachers' subject knowledge in mathematics and science has been extensively documented in the literature over the past two decades. Concern in mathematics has been documented by Ball, (1990); Frid, Goos & Sparrow (2006); Peard (2001); Peard (1998); Relich & Way (1992); White, Way, Perry & Southwell (2006); and Ryan & McCrae (2006). Results obtained from studies in science education indicate that practising teachers often possess naïve and fragmented views of science (Pomeroy, 1993) while similar concern about pre-service teachers has also been expressed (Shapiro, 1996; Taylor & Francis, 2001).

White et al. (2006) tested 78 pre-service teachers and reported that "overall achievement (in mathematics) was poor" (p. 43). Attempts in Australia to improve pre-service teachers' subject knowledge have been varied. In the State of New South Wales, for example, a prerequisite of Year 12 mathematics for teacher accreditation has been placed. However, White et al (2006) question the effectiveness of this noting that almost all the participants in their study met this requirement. Furthermore, an earlier study by the author of the present paper in Queensland found those students who had done only Year 10 mathematics achieved little differently in the Foundations unit from those who had done a Year 12 mathematics subject and that there was little difference in achievement between those students who had done an academic mathematics subject to those who had done a non-academic subject. He concluded "the selection of an academic course in Year 12 does not in itself mean a greater chances of success at the tertiary level, at least in primary teacher education" (Peard, 2004, p.424), and reported that in Queensland, and in other Australian States, higher achieving students are encouraged to take the academic option while lower achieving students are encouraged to take a non-academic option. In addition, there is evidence, both anecdotal and research, that many students begin teacher education displaying misconceptions, negative attitudes towards and apprehension of both mathematics and science (See, for example, Frid, Goos & Sparrow, 2006; Grootenboer & Lowrie, 2002; Kruger & Summers, 1998). Ryan & McCrea (2006) reported "significant proportions of cohorts on entry to initial teacher education have the same errors, misconceptions and incorrect strategies (in mathematics) as children." (p. 87). Kruger & Summers (1988) reported that primary teachers' misconceptions were common and over a decade later Taylor & Francis (2001) observed that teachers as well as children have misconceptions about primary science topics. There is general agreement from these findings of research into pre-service teachers' beliefs that these students' misconceptions are acquired during their school experiences. There is also evidence that negative attitudes contribute to poor classroom teaching which in turn contributes to poor pupil attitudes, beliefs and performance outcome (White, et al., p. 36).

Informal and anecdotal evidence gathered by the author during the implementation of the Foundations unit confirm these observations and are reported in this paper. White et al. (2006) support this author's view that "if these pupils go on to become teachers, a cycle of negativity may be created unless an appropriate intervention breaks the cycle" (p. 36). and confirm the consistent correlation of negative attitudes towards mathematics and achievement in the subject.

Ryan & McCrea (2006, p. 87) believe that it is the responsibility of the tertiary institute to make the content comprehensible to the student. White et al. (2006, p. 47) note that the best way to reach the required level of subject knowledge is via well constructed units in teacher education programs. Hence, it would appear that if change is to occur, it must come through suitable intervention at the tertiary level. However, most pre-service primary teacher education programs in Australia are able to allow only a limited time for the teaching of mathematical and scientific content. It is therefore important to make the most efficient use of the limited time available to improve the general mathematical and scientific competencies of pre-service primary teachers. In order to do this, we must first establish what constitutes quantitative literacy and how it can be best taught.

This paper will outline an attempt at appropriate intervention by the implementation of the integrated unit, Quantitative Literacy, which has been developed by the author. Historically, much mathematical and scientific knowledge has been developed in response to social need and the unit consists of an integration of topics from mathematics and science in which all content is presented in a social and historical contexts. Research into the effectiveness of this approach is on going and some results of earlier research are referenced here.

Dimensions of the Unit

There is no one single definition of or universal agreement on what constitutes quantitative literacy. Clearly it includes "numeracy" however attempts at defining even this simpler term have been fraught with difficulty (Willis, 1989). The authors of "A National Statement on Mathematics for Australian Schools" (Australian Education Council, 1991) made the point that while the desirable characteristics of a numerate person can be identified, it is much more difficult to say precisely what numeracy is.

Kemp & Hogan (2000) define numeracy:

> Numeracy is having the disposition and critical ability to choose and use appropriate mathematical knowledge strategically in specific contexts.

Willis (1989) in recognising the inadequacy of any single definition goes on to say:

> A numerate person would use a blend of mathematical, contextual and strategic knowledge when required to use mathematics in a practical setting. (p. 34)

In terms of scientific content, when we consider what is “essential”, what is “important” and what is “desirable”, there is simply too much factual information for any educators to come to agreement. Rather we need to consider how scientific knowledge is acquired. Mc Donald (2007) gives comprehensive account of the acquisition of scientific knowledge over the last few decades, however none of the numerous studies cited include the use of history to achieve this end.

In this Unit historical topics are used to show how beliefs and understandings change as they develop and students are encouraged to constantly question their own beliefs and understanding of quantitative (mathematical and scientific) ideas. It is the opinion of the present author that in order to develop effective quantitative literacy attitudes and misconceptions reported in the literature must first be overcome and that this is best accomplished by presenting all subject content in relevant social and historical contexts.

There is a common misconception that mathematics and science are culture free subjects. Bishop (1998) gives a thorough refutation of this notion. In this Unit, mathematical and scientific thinking, beliefs, and problem solving are examined only in social and historical contexts. The decontextualised presentation common in many mathematics courses is avoided.

According to David Suzuki, “Today the most powerful force affecting our lives is not politics, business, celebrity or sports despite the coverage they receive in the media. By far the greatest factor shaping the world is science….Without a basic knowledge of scientific terms and concepts and an understanding of how science differs from other ways of knowledge we cannot find real solutions to such issues as global warming, toxic pollution, species extinction, overpopulation, alienation and drug abuse.” (Suzuki, 2006, p. 324)

In recognising that the body of quantitative knowledge is vast, we select relevant topics in order to illustrate how quantitative knowledge has been constructed in response to social need. We examine beliefs about the nature of science and mathematics, their roles in society, and the contribution they have made to the growth of knowledge. Included are: myths and misconceptions about mathematics and science; the scientific method and the formation of hypotheses; the nature and role of problem solving; induction and deduction in mathematics and science and the role of patterning and making generalisations.

Delivery of the Unit

The unit is delivered as a large group (300+) lecture of 1-2 hours followed by a small group (25) tutorial of two hours. Tutorial activities incorporate reflection on lecture topics, practical and problem solving activities. Students keep a reflective journal of their reaction to the lecture and tutorial activities. Journal entries are made each week. These consist of an account of problem solving activities, data from practicals and analysis, and a reflection on the lecture topics. Assessment criteria of journals include evidence of reflection on and personal reaction to the lecture topics. This encourages honest responses when asked, for example, to answer in their journal “What did you learn from the lecture? What parts of the lecture did you find most interesting?" As a result of many such entries the prevalence of misconceptions becomes apparent.

Throughout, a problem solving approach is taken. Students in the unit come from a variety of mathematical backgrounds and little pre-requisite knowledge is assumed. Rather mathematical and scientific knowledge is to be constructed via the examination of the topics within the contexts described.

Common Misconceptions

From an examination of journal entries over a number of years, common views and misconceptions of mathematics reported include:

- Only very intelligent people can understand it. Some people can't do it at all.
- Mathematics never changes.
- Mathematics requires the memorisation of lots of rules and formulae.
- There's no room for opinions in mathematics, everything is either right or wrong, true or false.

Misconceptions in science reported in the literature (Taylor & Francis, 2001) are also evidenced here. These include:

- The earth is closer to the sun in summer (common),
- The sun is always directly overhead at 12:00 noon.
- The phases of the moon are caused by shadows cast on its surface by other objects in the solar system.
- The moon emits its own light (rare)
- The moon has a dark side
- The terms AD and BC have been in use since 1AD
- Columbus was the first to show the earth was round (common)

Of the moon misconceptions, it is interesting to note that many who held these views had completed successfully advanced senior mathematics and /or science subjects at high school. It is quite disturbing that these same students would have studied extensive calculus. Calculus was developed by Newton, largely to enable him to explain planetary motion and the functioning of the physical world. Yet here we have students studying calculus who have little comprehension of that very physical world. We might ask how widespread this phenomenon is and what does it imply about the whole nature of senior secondary education. This is included in the conclusion as a recommendation for further research.

Some Bases for our Beliefs and Reasoning

When asked to write about the basis for their mathematical and scientific beliefs the vast majority answer: the authority of the teacher and the text books. Students are generally unaware of how beliefs, mathematical, scientific and everyday have developed. Historical topics are used to show this and that some beliefs different bases; authority, personal beliefs and faiths, beliefs arrived at inductively and deductively, and beliefs without foundation.

We recognise the need for beliefs based on authority; young children must rely on their parents authority as to what is good for them, how to cross the road, what to eat, etc. Pupils rely on the authority of teachers and texts. Citizens rely on the authority of politicians, the media, newspapers, magazines, books etc. However, not all of these authorities are always reliable, and history proves us with an excellent avenue to illustrate this. For example, the most common of these is the misconception that Columbus was the first to show that the earth is round. Most are unaware that the Greeks not only knew the shape of the earth some 2000 years earlier but also that Eratosthenes (c. 200BC) had accurately measured the circumference. They are mostly unaware of the evidence known to most earlier societies; the fact that you can see further from higher elevations, that as a ship disappears over the horizon its sail disappears last, and the earth's shadow on the moon during an eclipse. (A Unit activity models the method Eratosthenes used to measure the circumference of the earth, See Appendix).

We find also that many beliefs without foundation are common. These include misconceptions and prejudices and belief in things for which there is no evidence. Of the latter, astrology is the most widespread. In the unit we examine the historical nature of such beliefs. For example, in ancient Egypt, the year started when the star Sirius was first observed to rise in the morning sky. This was always followed by the flooding of the Nile. Thus they believed the *cause* of the flood was the rising of the star. They then looked for other natural occurrences and related them to star positions. This is an example of the sort of event of the time that led to the belief in astrology. In these situations the Egyptians were reasoning *inductively* in the absence of other information. We know today that Sirius does not cause the Nile to flood and that no other stars have any influence on what happens on earth.

The importance of *deductive* reasoning in establishing quantitative beliefs is illustrated by examples such as that the Egyptians (4000 years ago) knew the formulas for areas and volumes, but they didn't know *why* they worked. For example, they observed that if you made a pyramid of any size, its volume was always 1/3 that of the surrounding prism. The volume of a cone was always 1/3 that of the surrounding cylinder. It was the Greek mathematicians who showed deductively *why* the formulas worked. *Induction and exploration* suggest conclusions which later may be proved. If only the final proof is considered, the process of exploration (including mistakes) is generally lost. Unfortunately much "school" mathematics and science presents only the final results and ignores the procedures that led there. A knowledge of the history of the development helps us understand these procedures and is therefore include in the Unit.

Examples of historical topics employed

Some related Tutorial activities are shown in the Appendix. A few brief examples of topics selected are shown below.

Astronomy. We start with the historical need to know where to locate the planets, the moon, and the stars, and to be able to keep track of their positions and movements. We examine the Greek model of the universe of two spheres with the earth at the centre proposed by Aristotle and which made sense at the time, followed by the contribution of Copernicus and Galileo who recognised that Copernicus' model made more sense of the data and was a strong proponent of it. Galileo pointed his new telescope at Jupiter and saw for the first time ever the moons of Jupiter going around the planet. Here was first hand evidence that not *all* celestial objects revolved around the earth as Aristotle had claimed and as the church believed. He also observed Venus and saw that it had phases (like the moon has) when observed from earth. These were consistent with its moving around the sun, not the earth.

Navigation. The determination of latitude and longitude, particularly the history of the difficulties associated with the determination of longitude. We conclude by asking "If the Greeks knew the shape and size of the earth in 200BC, where did the nonsense about Columbus (1700 years later) come from?"

Measurement of time and the history of the calendar: We start with noting that most societies were able to arrive at a figure of 365 as the nearest whole number, and that they realised that this figure was too low so that additional days had to be added. From the ancient Egyptians who were the first that we know of to have an accurate calendar about 6000 years ago to the Roman, Julian, Gregorian and the modern calendar.

Gravitation (Galileo's inclined plane experiments and pendulum)

The history of measurement from early time to the Metric System.

Numeration. An examination of Egyptian, Greek, Roman, Babylonian, Mayan, Aztec, Chinese, Hindu-Arabic systems.

Computation: Various methods of computation developed in the different numeration systems are examined. Computations in the many systems done on the abacus are compared with "school taught" algorithms which have their origin in the algorithms that developed with the spread of the Hindu Arabic system. We note with interest the opposition to the use of written algorithms over the abacus and compare it with opposition to the calculator over written algorithms today.

The measurement of chance and the history of probability. We note that although ideas about probability have been recorded for thousands of years, a formal study of probability is very recent and that the study of probability is historically a recent development in the field of mathematics having its origin in the analysis of gambling situations in the 17th century, extending to the need for insurance in the 18th century as trade by shipping expanded, and developing into a recognised branch of mathematics subsequent to this, with applications in business, finance, science and medicine. Initial opposition to the study of probability came from organised religion who believed that everything was determined by God's will. Probability did not exist. Until the latter half of the 20th century, the study was confined to the tertiary level. Now however, it is part of both the primary and secondary school curriculums. The reasons for this are examined.

Research on the effectiveness of the approach

Research by Peard & Pumadevi (2007) used student responses to Discussion Forum items as well as Journal entries in a study into the effectiveness of the unit. This research reports that in Malaysia, where the unit has been taught at two tertiary institutes, as well as at QUT, the use of historical topics and a problem solving discussion based approach both contribute to better understanding of concepts, the development of social skills and most importantly improves attitudes to the subjects. Peard & Pumadevi (2007) concluded that the implementation of the unit is consistent with the recommendations of recent research in the field. Research at QUT is ongoing.

Conclusions

The level of quantitative literacy of students entering pre-service primary teacher education is a cause for concern as many students enter these programs with misconceptions about the nature of such knowledge. It is recommended that further research be continued to determine which misconceptions are common and the degree to which they are held. It is also recognized that it is the responsibility of tertiary institutions to make suitable intervention programs within teacher education courses at the tertiary level. Such a program requires the identification of students' needs in this field and requires the consideration of what constitutes appropriate quantitative knowledge for them. The first objective of any such program must be to break the negativity cycle described in the literature. The conclusion that the program described at QUT incorporating the use of historical topics in a problem solving based approach is largely successful in doing this is supported by the research of Peard & Pumadevi (2007) reporting new positive attitudes to the subjects, an appreciation of the importance of quantitative knowledge and improved understanding of the nature of quantitative reasoning.

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Appendix

Some Unit tutorial activities incorporating the use of historical topics

1. Measuring the earth - Eratosthenes (c. 200BC)

- Use one of the styrofoam balls and two toothpicks. Attach two toothpicks to each ball and label them A for Alexandria and S for Syene.
- Shine a torch on the ball so that it's shining directly on the toothpick labelled S and does not cast a shadow.
- Mark the length of the shadow at toothpick S.
- Record the experiment for your journals. Use a protractor and compass so that diagrams are labelled accurately.
- Use the following proportion to calculate the circumference of each ball.

$$\frac{\text{angle}}{360°} = \frac{\text{distance}}{\text{circumference}}$$

- Measure the actual circumference of each ball using string. How close is the actual measurement to the calculated measurement for each ball? What might have affected the accuracy of your results?
- Draw a diagram of your experiment and use your methods and results to explain how Eratosthenes used geometry to determine the size of the earth. What assumptions did he have to make? Based on your experiment, discuss why his calculations may not have been correct (about 50 words).

2. In 1772 a German astronomer named Bode discovered a pattern in the distances of the planets from the sun. At that time only 6 planets were known. Using the distance from the earth to the sun as 10 arbitrary units, Bode constructed the following table:

Planet	Distance from sun	Bode's number
Mercury	3.9	4 (4+0)
Venus	7.2	7 (4 +3)
Earth	10.0	10 (4 +6)
Mars	15.2	16 (4 +12)
XXXX	XXX	XXXXXX
Jupiter	52.0	52 (4 + 48)
Saturn	95.4	100 (4 +96)

Bode predicted a planet would be found between Mars and Jupiter.

Why? What type of reasoning was he using?
What would be the Bode number of this planet?

What was later discovered to lie in this gap between Mars and Jupiter?
Bode also calculated a number for the next planet beyond Saturn.
What is this number?
In 1781 William Herschel discovered a planet at a Bode number distance of 192 units.
What was it named?

3. (a) Add the following Mayan numbers (Work with Mayan symbols. Do NOT convert to base 10).

(a)

<u>...</u> + <u>....</u>

(b)

.
0 + ...
—

(b) Draw a diagram of a Roman abacus. Show how you would do the following additions on the abacus (use counters)

(a) XXIII + XI
(b) XVIII + XIII
(c) MCCCLXXVIII + DCCLXV

4. Why do we have leap years? How they are determined?
If Jan 1st this year was on a Monday what day of the week will it fall on next year?
What day in 2016?
Construct a calendar in which the day of the week and date are constant from year to year.